Influence of uncertainties on the stability of rock cavities under gravity and seismic loads

F. de Silva[1], A. Flora[2], M. Nuzzo[3], F. Silvestri[4]

[1]*Associate Professor, University of Naples "Federico II", Naples, Italy, email:* filomena.desilva@unina.it

[2]*Full Professor, University of Naples "Federico II", Naples, Italy, email:* alessandro.flora@unina.it

[3]*Superintendent of Archaeology, Fine Arts and Landscape for the provinces of Caserta and Benevento, Italy, email:* mariano.nuzzo@cultura.gov.it

[4]*Full Professor, University of Naples "Federico II", Naples, Italy, email:* francesco.silvestri@unina.it

ABSTRACT

The paper evaluates the stability under gravity and seismic loads of anthropic cavities, which have been extensively excavated in the volcanic tuff for sheltering, storage and extraction purposes in the subsoil of several historical centers throughout the provinces of Naples and Benevento (Southern Italy). The safety conditions were checked through stability charts, expressing the ratio between the rock uniaxial compression strength and the overburden vertical stress as a function of i) the cavity width, normalized to the roof thickness, and of ii) the peak ground acceleration. Four different stability zones are distinguished in each chart through boundary curves associated with three different probabilities of failure. The latter have been obtained by the authors through a reliability-based approach that considers the uncertainties due to the spatial variability of the rock uniaxial compression strength and the presence of discontinuities under gravity loads, as well as the uncertainty due to the input motions in seismic conditions. The predictions were compared with those obtained from the application of a more traditional deterministic approach, revealing that neglecting uncertainties may lead to an excessively unconservative assessment.

Keywords: historical centre, anthropic cavity, safety assessment, stability chart, large-scale approach.

INTRODUCTION

The presence of anthropogenic underground environments represents a distinctive feature of the subsoil of many urban areas in Southern Italy, particularly throughout the provinces of Naples, Caserta and Benevento, in the Campania region. These cavities, excavated in different historical periods and for various purposes — from the extraction of construction materials to water collection, food storage, and defensive functions — today constitute an essential component of the cultural and geotechnical landscape of several historical centers. The interaction between subsoil and urban morphology has produced a complex distribution of cavities, cisterns and tunnels which, while being of remarkable historical and technical value, raise critical issues for the safety of the population. In many cases, such underground structures are directly connected to the above-ground historic built environment, thus requiring an integrated approach to assess their static and seismic stability. In particular, the integration of geomatic and geological surveys with geotechnical and structural investigations and analyses, combined with an interdisciplinary collaboration among geologists, engineers, archaeologists and conservators, is essential to develop reliable predictive models and shared protocols for preventive countermeasures against their instability.

In the area of Naples, the widespread presence of soft volcanic rocks such as Neapolitan Yellow Tuff and Campanian Ignimbrite has facilitated, for centuries, the excavation of underground cavities, often later reused as cisterns or shelters. Similarly, in the Caserta area the tuffaceous formations of Maddaloni and Mount Tifata gave rise to a vast network of cavities beneath the historic centers of Caserta Vecchia, Sant'Agata de' Goti and Piedimonte Matese. In Benevento, instead, cavities have been mainly excavated in calcarenites and conglomerates, for storage or defensive purposes.
Analogous practices were developed in Apulia and Latium, where the presence of calcarenites and lithoid tuffs, respectively, favored the creation of similar underground structures — evidence of a widespread constructive tradition and a millenary relationship between human settlement and the subsoil. Nevertheless, the Campania region stands out for the extension and complexity of its underground systems, which simultaneously involve geological, archaeological and urban factors.
The aim of this paper is the fine tuning of simplified but reliable tools for a preliminary assessment of the stability of cavities in Naples, Caserta and Benevento areas. This provides the basis for a unified conservation strategy for the underground heritage of Campania, founded on probabilistic assessment methods and innovative monitoring tools. Understanding the static and dynamic mechanisms of these environments has not only engineering significance, but also historical, artistic and cultural importance: anthropogenic cavities are a tangible record of the underground city and its transformations. The integration of geotechnical research and heritage conservation thus represents an advanced frontier of preservation, capable of combining safety, enhancement and knowledge of the territory within a truly sustainable framework.

METHOD OF ANALYSIS

Stability under gravity loads

A useful straightforward tool to assess the stability of rock cavities is the chart in Figure 1a, in which a power law function bounds the failure conditions of the roof of an anthropic rock cavity under gravity loads. Such curve has been derived through numerical analyses by de Silva et al. (2023), but other analytical expressions exist in literature, e.g. by Evangelista et al. (2000) or Suchowerska et al. (2012). By knowing the ratio between the span and the thickness of the roof, *L/t*, the cavity is expected to be safe, if the ratio between the rock uniaxial compressive strength, σ_c, and the vertical stress acting in the roof, σ_v, exceeds the value predicted by the boundary curve.

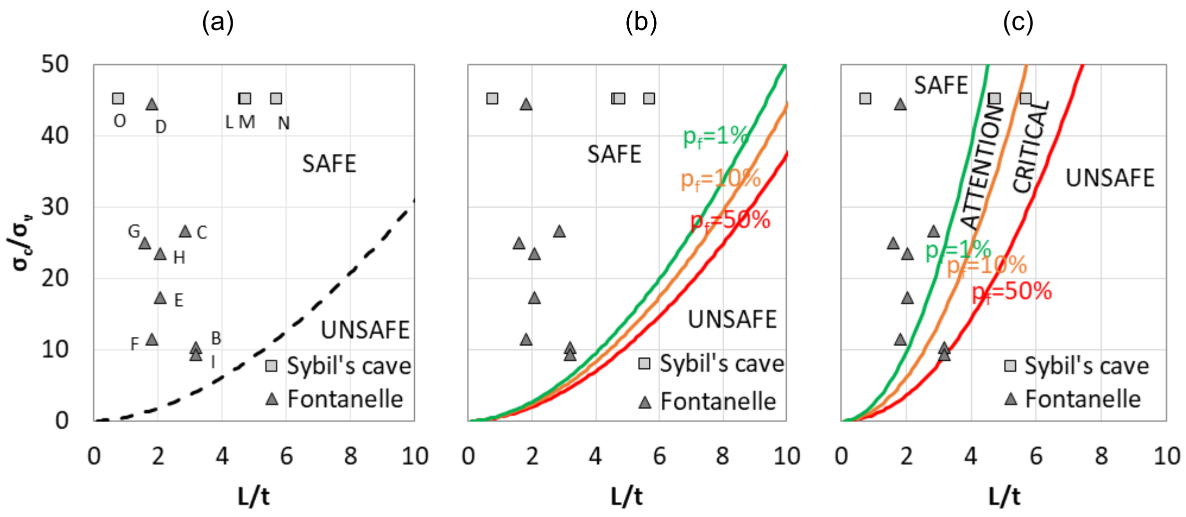

Figure 1. *Scheme of the deterministic (a) and reliability-based stability charts accounting for the variability of rock strength (b) and presence of discontinuities (c). Markers indicate several cross sections of case studies used to validate the approaches, as discussed in the following.*

The uncertainties due to the spatial variability of rock strength and the presence of discontinuities have been considered by de Silva et al. (2023). This variability was introduced in the numerical analyses of cavities, by changing the uniaxial compression strength between 1 MPa and 5 MPa within the model or by modelling the discontinuity at different depths, with different values of the strength parameters and in any case lower with respect to that of the rock.

For a given value of *L/t*, different values of the critical ratio σ_{cmin}/σ_v were found from all the analyses, typically resembling a log-normal distribution. In such a case, the inadequacy of a cavity to support the applied loads can be quantified through the probability, p_f, that σ_{cmin}/σ_v is higher than σ_c/σ_v. This allows deriving curves of σ_c/σ_v - *L/t* for fixed values of p_f, that can be exploited as thresholds for the stability assessment of the cavity roof. In this study, probability of 1%, 10% and 50% were selected, leading to the curves in Figure 1 when either the rock strength (b) or the position of rock joints (c) is the unknown influencing factor. They bound four zones in the stability chart (Figure 1c), that can be associated with relevant actions to be undertaken to safeguard their integrity:

SAFE ZONE: the cavity roof is expected to be safe;

ATTENTION ZONE: the cavity roof needs to be monitored;

CRITICAL ZONE: the stability of the cavity roof needs to be assessed by more refined tools;

UNSAFE ZONE: the stability of the cavity roof needs to be assessed by more refined tools and interventions may be required to ensure the stability in the meantime.

. Stability under seismic loads

The stability of cavity roof under seismic loads has been studied by Fabozzi et al. (2024) assuming the cavity roof equivalent to an idealized fix-ended beam, characterized by the same values of *L*, *t* and σ_c so that the resisting moment of a section with unitary out-of-plane width is:

$$M_R = \sigma_c \frac{t^2}{22} \tag{1}$$

where the tensile strength is set equal to 1/10 σ_c.

The beam is considered loaded by a uniformly distributed overburden stress equal to σ_v and by a seismic-induced rotation of the extremities, θ. By exploiting the well-known 1D wave equation of motion into a pseudo-static approach, the rotation results dependent on the peak ground acceleration on surface, *PGA*, on the shear wave velocity of the rock, V_S, and on the roof depth (namely, *t* plus the thickness of the covering soil, *p*) through the following expression:

$$\theta = \frac{PGA}{V_S^2}(t+p) \tag{2}$$

The equilibrium of the acting and resisting moment leads to the following expression:

$$\frac{\sigma_c}{\sigma_v} = 22\frac{t}{L}\left(\frac{1}{12}\left(\frac{L}{t}\right)^3 + (1+\nu)\frac{PGA}{g}\right) \tag{3}$$

in which *g* is the gravity acceleration and ν the Poisson's coefficient.

Figure 2a shows the resulting stability chart for ν=0.2 and *PGA*=0.17g.

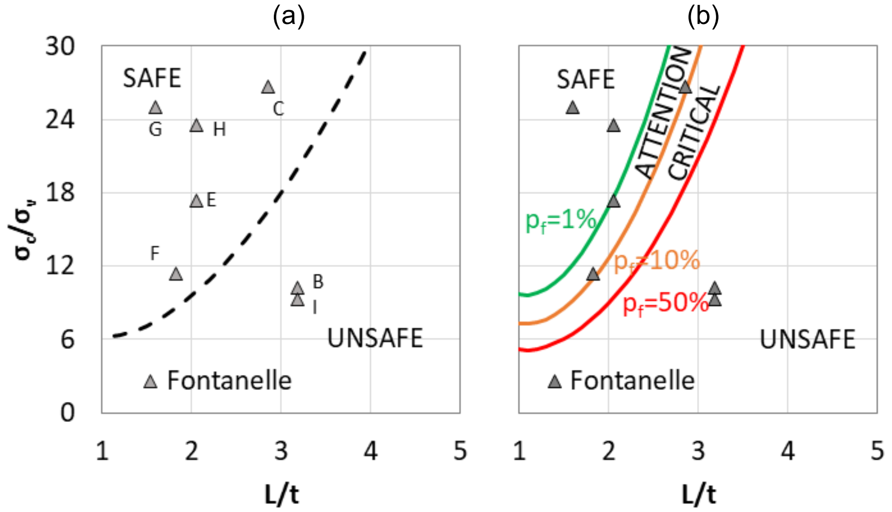

Figure 2. *Deterministic (a) and reliability-based (b) stability charts considering the variability of cavity geometric features, rock uniaxial compression strength and seismic input motions.*

Note that such chart refers to the achievement of the resistance in the extreme sections of the beam, not to the full collapse of the roof. This aspect, jointly to the effect of the PGA, is the reason why they appear much more severe than the chart in Figure 1a.
Based on the results of several numerical analyses, Fabozzi et al. (2024) derived the chart in Figure 2b considering the uncertainties associated with the cavity geometric features, the rock uniaxial compression strength and the seismic input motion. Again, different values of the probability of failure can be set to bound zones characterized by different safety conditions.

VALIDATION THROUGH COMPARISONS WITH MORE REFINED APPROACHES

The stability predictions obtained by the charts shown in Figures 1 and 2 were checked against the results of refined numerical analyses executed on two case-studies of historic monumental cavities in Naples. The first one is the Fontanelle cemetery, an ancient 'room and pillar' cave excavated inside a 40m high hill, constituted of the local Neapolitan Yellow Tuff. The rock mass is covered by a pyroclastic silty sand layer, commonly named Pozzolana, with a variable thickness from 0 m to 20 m, as clearly visible along the cliff faces. The hill is crossed by two 80 m long corridors along the N-S direction and by a shorter NW-SE aisle, respectively named "Preti nave", "Appestati nave" and "Pezzentelli nave", since clergymen, victims of plague and poor children were buried there from the XVI century. The intersection of the three main naves with some secondary E-W rooms originates the nine pillars shown in Figure 3a-b. The current ground level does not correspond to the bottom of the excavation. In fact, geophysical tests confirmed that the bottom of the excavation is almost flat and 9 m deep (Scotto di Santolo et al., 2015).
The numerical model of the Fontanelle cemetery shown in Figure 3a-b was generated through the three-dimension finite element software FLAC3D (Itasca, 2015). The high modelling capability of the software allowed to import the cloud points surface obtained from a laser-based survey (de Silva et al., 2018) to correctly reproduce the complex geometric features of the cave. A Mohr-Coulomb constitutive model was assigned to all materials with the properties shown in Figure 3a. The small-strain shear modulus, G_0, of tuff and debris sublayers were computed from the shear wave velocities measured on-site. All the unit weights, γ, and the Poisson's coefficient, v, as well as the mechanical parameters of the pozzolana layer are the typical values inferred from the experimental data collected at the Federico II university during the numerous research activities on such materials (e.g. Picarelli et al., 2007). The friction angle, φ, and the cohesion, c, were assumed constant for the pozzolana and debris materials, while a reduction of the cohesion until a value c=423kPa with increasing shear strain was introduced for the tuff to simulate the reduction of uniaxial compression strength, σ_c, revealed by laboratory tests. The tensile strength, σ_t, was limited to the to 1/20 σ_c for the tuff formation and 1/10 σ_c for the other soil layers.
The second case study, in Figure 3 b-d, is the cave of the Greek prophet Sybil, that is the most famous monument in the Archaeological Park of Cuma, in the active volcanic area of the Phlegrean Fields (west of Naples). Archaeological research interprets the monument as a military tunnel dug into the Neapolitan Yellow Tuff parallel to the coast for a length of about 180 m between the end of the IV and the beginning of the III century BC. The initial trapezoidal shape was modified by Romans who lowered the ground level. On the wall oriented towards the coast, there are nine smaller tunnels, six of which are open to the front side. On the opposite wall, there is a rectangular room with three additional smaller rooms. At the end of the tunnel there is the main room, where it is said that the Sibyl prophesied.
Recently, a laser scanner survey of the cave and of the topographic surface was carried out, which, were imported into FLAC3D to generate the numerical model of the portion closer to the main room, shown in Figure 3 b. The survey and the outcrop of the tuff roof on the lateral side have shown that the thickness of the tuff vault is very small at the end of the gallery, exactly where the main room is located. Uniaxial compression tests performed on specimens obtained from tuff blocks found in the central area of the cave led to σ_c about 5 MPa on dry specimens and 4.3 MPa on saturated specimens, but the presence of numerous discontinuities detected on site reduces the strength of the rock mass. This was estimated to be 750 kPa by Diletto (2021), calibrating the Hoek and Brown (1980) criterion for a geological strength index GSI=74, resulting from the discontinuity analysis. Figure 3 b shows the parameters adopted in the numerical model considering the Mohr-Coulomb resistance criterion equivalent to that of Hoek and Brown in the range of 10 kPa$<\sigma'_3<$150 kPa. Finally, a layer of degraded material with a thickness of 1 m was modelled, to consider the reduced cementation of the tuff roof.
Both models were analyzed by firstly executing the lithostatic equilibrium of the hill and then by simulating the excavation through removal of the tuff rock elements.
Figure 3 c and d show the plastic points in tension of the cavity roof at the end of the analysis. In both cases, the caves are stable, except for some detachment of block expected to occur due to the local

attainment of the tensile strength. The latter results close to section B, C F in the Fontanelle cemetery and in correspondence of section N for the Sybil's cave. This is in accordance with the prediction of the chart in Figure 1c accounting for the presence of joints and the strength variability, where the section B and N fall in the critical stability zone, while C and F are very close to the attention zone. Damaged conditions are predicted by the chart even for section I, for which the numerical model yields only a couple of plastic points.

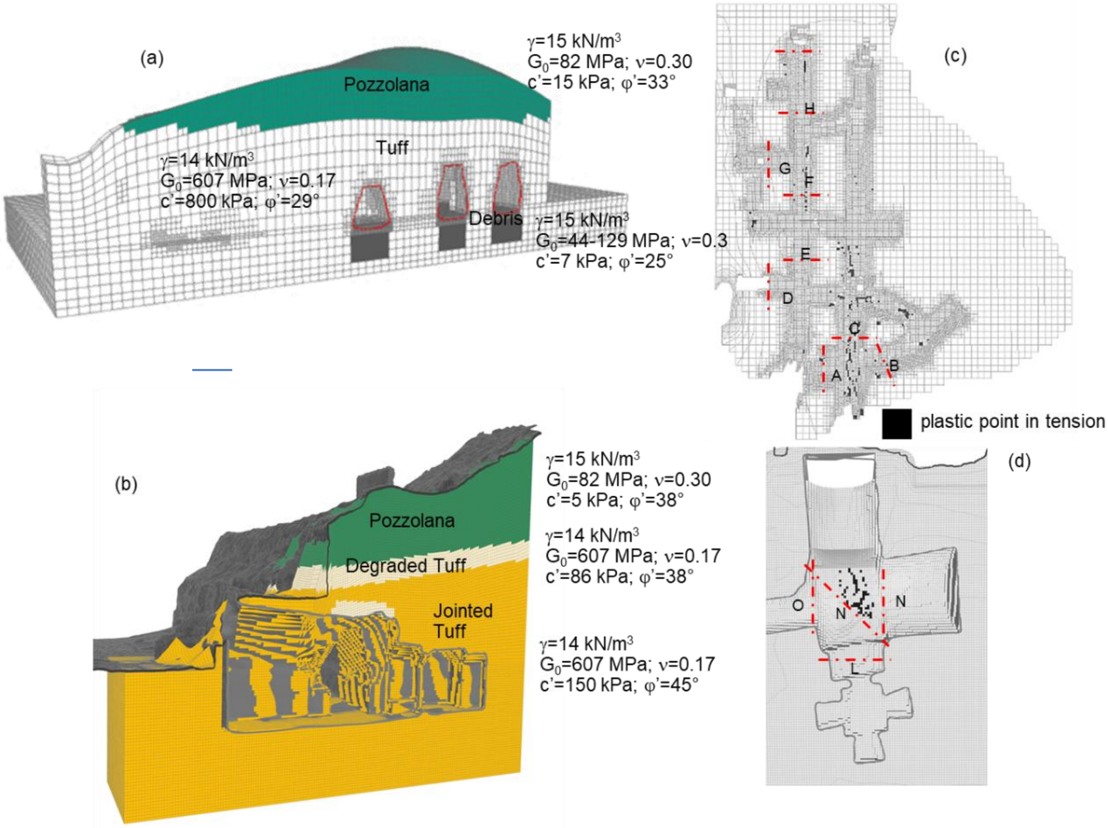

Figure 3. *3D numerical models (a-b) and plastic points in tension (c-d) of Fontanelle cemetery (a, c) and Sybil's cave (b, d). The red dashed lines are the cross sections used to validate the stability charts in Figure 1.*

The Fontanelle cemetery roof under seismic actions is expected to be unsafe in sections B and I and critical in sections F and C according to the chart in Figure 2b. This condition is confirmed by de Silva and Scotto di Santolo (2018) who executed nonlinear dynamic analyses under the strongest historical earthquake even occurred in Naples, characterized by a PGA comparable to 0.17g. The outcomes interpreted through a probabilistic approach leads to a probability of damage equal to one for sections C and I and around 0.8 for section F. Table 1 summarizes the safety conditions assessed through the different approaches.

Table 1. *Classification obtained from the charts and damage resulting from numerical analyses in terms of plastic points. Green, yellow, orange and red colours correspond to safe, attention, critical and unsafe safety conditions predicted through charts, while x indicates that plastic points resulted in the section from numerical analyses.*

	Fontanelle cemetery																		Sybil cave			
	Gravity loads									Seismic loads									Gravity loads			
Section	A	B	C	D	E	F	G	H	I	A	B	C	D	E	F	G	H	I	L	M	N	O
Deterministic	green	green	green	green	green	green	green	green	green	green	red	green	green	green	green	green	green	red	green	green	green	green
Probabilistic	green	orange	green	green	green	green	green	green	orange	green	red	orange	green	green	orange	green	green	red	yellow	yellow	orange	green
Numerical		x	x			x						x			x			x			x	

APPLICATIONS AT TERRITORIAL SCALE

All the mentioned approaches have been applied to the cavities dug in the historical town of Sant'Agata de Goti, in the south of Italy and in the city of Naples. They are affected by seismic hazard, with PGA=0.17 g expected to occur on a stiff rock outcrop with a return period of 475 years, as shown in Figure 4a.

S.Agata de' Goti rests on the flat summit of a hill delimited by subvertical cliffs as high as 50 m. The hill is mainly formed by the Campanian Ignimbrite, a volcaniclastic formation erupted in the Campi Flegrei about 39,000 yrs BP (De Vivo et al., 2001), covered by a layer of pyroclastic soils and made ground. The formation is affected by intersecting joint systems, forming rock columns or slabs visible along the cliff. The shallowest few meters of the Campanian Ignimbrite are lithified, with a mean uniaxial compression strength σ_c around 3 MPa. Due to its resistance, such material has been extracted in the past to construct the above ground buildings. The quarrying activities left more than 160 anthropic cavities in the area of 1 km^2 of the oldest settlement.

The shallowest subsoil of Naples is characterized for its largest part by the abovementioned tuff, characterized by a mean σ_c = 3 MPa, that varies with the rock porosity and the included lithic fragments. In many areas of the town, tuff outcrops or is at a rather shallow depth, hence it has been excavated since very old times leaving a dense network of more than 900 cavities underneath the city. The typical cross - section is characterized by slightly inclined walls and almost flat ceilings often crossed by horizontal syngeneic joints, locally called *suoli*.

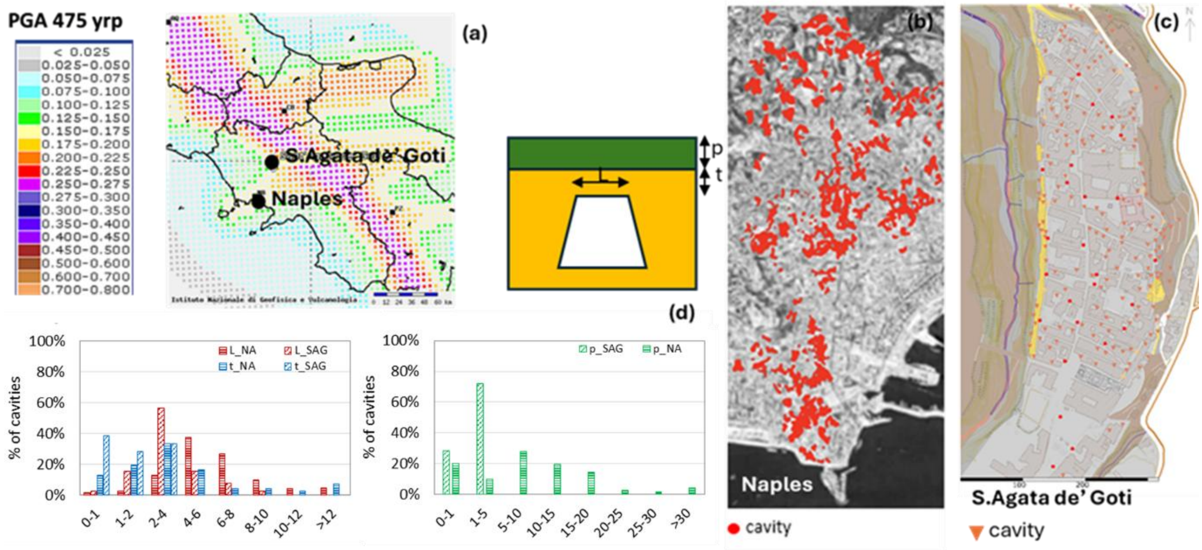

Figure 4. *Location of S. Agata de' Goti and Naples on the seismic hazard map of Italy (a); existing cavities in Naples (b) and S. Agata de' Goti (c) and statistics of the geometric features in meters of the detected ones (d).*

Figure 4d shows the statistics of *L*, *t* and *p* detected for 39 cavities in S. Agata and 123 in Naples. The statistics show that cavities in S. Agata are typically shallower (lower *t* and lower *p*) and narrower (lower *L*) than those in Naples. In S. Agata most cavities were quarried exactly in the same area of the above ground to be part of it. In fact, the most frequent *L* ranges between 2 and 4 m, that is the typical span length of masonry buildings. Conversely, in Naples, the largest cavities were used as quarries for more than one building and later enlarged and arranged to host different functions.

Given the detected *L/t* ratio and the measured σ_c, safety was assessed by considering the σ_v at the depth of the cavity ceiling calculated by setting the unit weight of cover and rock equal to 16 kN/m^3 incremented by a uniformly distributed load equal to 100 kPa, due to the above ground buildings. A *PGA* = 0.27g was set for S. Agata, because Piro et al. (2019) calculated a *PGA* amplification factor around 1.6, due to the topographic irregularity of the cliff joint to the stratigraphic amplification of the seismic waves in the softer shallow soils. The same *PGA* can be reasonably assumed as a mean value for Naples, but additional studies are necessary, due to the expected largest variability of the cavity depth and of the subsoil conditions.

Figure 5 shows the percentage of cavities falling in the four different safety zones under gravity and seismic loads for both sites. Under gravity loads and through the deterministic approach, almost all cavities are stable in both sites. The assessment is quite the same, if the variation of the rock strength is considered, but is significantly worsened by the presence of discontinuities. In fact, the safe cavities reduce to 70% in S. Agata and 50% in Naples, and 25% of them are completely unsafe in the latter site. Under seismic loads, the percentage of unsafe cavities in S. Agata is not negligible, while most of the Neapolitan cavities are unsafe due to their larger size. Again, the probabilistic approach leads to worse conditions.

Conclusions

The paper investigated how the safety under gravity and seismic loads of the roof of cavities dug in soft rocks is influenced by the uncertainties on the rock strength and loading actions, usually overlooked in simplified approaches applicable in large-scale assessment. Reliability-based stability curves have been obtained considering different probabilities of failure for the possible random variability of rock strength, horizontal joint location in the roof and seismic actions. The predictions of the charts have been validated through comparisons with the outcomes of advanced 3D numerical analyses executed on two very famous monumental cavities in Naples.

The outcome of the application to the case studies of Naples and S. Agata de' Goti reveals that neglecting the uncertainties on rock strength and integrity, as well as on the seismic actions can be dangerously unconservative, even in the preliminary assessment of safety conditions.

These conditions highlight the urgent need for systematic mapping and continuous geotechnical monitoring of the underground cavities, both for public safety and for the preservation of monumental heritage.

In this vein, the predictions of the proposed charts should be interpreted as preliminary or screening-level assessments for prioritization, highlighting the cases in which detailed site-specific evaluation are necessary.

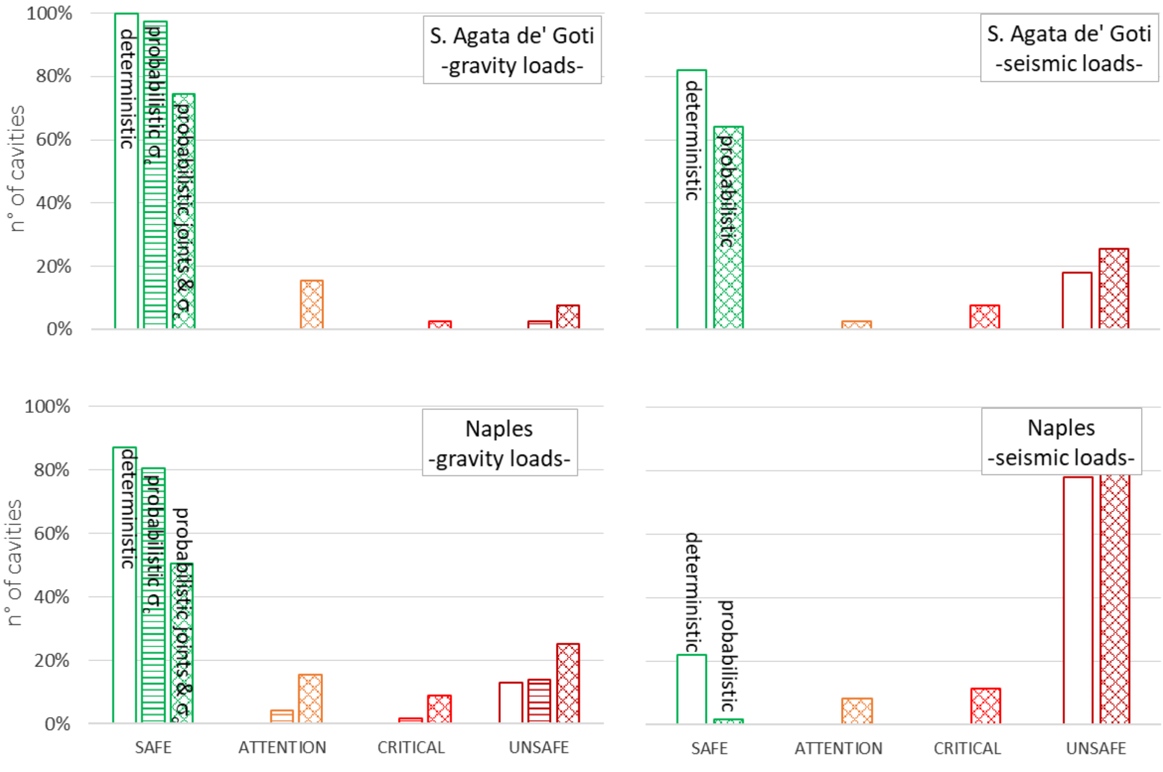

Figure 5. *Safety conditions of cavity roofs in the historical centers of S. Agata de' Goti and Naples under gravity and seismic load, by using deterministic and probabilistic approaches.*

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